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Jeffrey Hilton Miles
Lewis Research Center, Cleveland, Ohio

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Collective interaction in a linear array of supersonic rectangular jets: a linear spatial instability study

Jeffrey Hilton Miles
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, OH 44135

Abstract

A linear spatial instability model for multiple spatially periodic supersonic rectangular jets is solved using Floquet-Bloch theory. It is assumed that in the region of interest a coherent wave can propagate. For the case studied large spatial growth rates are found. This work is motivated by an increase in mixing found in experimental measurements of spatially periodic supersonic rectangular jets with phase-locked screech and edge tone feedback locked subsonic jets. The results obtained in this paper suggests that phase-locked screech or edge tones may produce correlated spatially periodic jet flow downstream of the nozzles which creates a large span wise multi-nozzle region where a coherent wave can propagate. The large spacial growth rates for eddies obtained by model calculation herein are related to the increased mixing since eddies are the primary mechanism that transfer energy from the mean flow to the large turbulent structures. Calculations of spacial growth rates will be presented for a set of relative Mach numbers and spacings for which experimental measurements have been made. Calculations of spacial growth rates are presented for relative Mach numbers from 1.25 to 1.75 with ratios of nozzle spacing to nozzle width ratios from $s/w_N = 4$ to $s/w_N = 13.7$. The model may be of significant scientific and engineering value in the quest to understand and construct supersonic mixer-ejector nozzles which provide increased mixing and reduced noise.

I. Introduction

Interest in proving the economic and environmental feasibility of a high-speed civil transport has stimulated studies of mixing enhancement in lobed mixer-ejector nozzles. By enhancing mixing the ejector length can be reduced with the same amount of noise suppression. In order to obtain information on such flows simpler configurations are studied. In particular, a simple mixer nozzle configuration consisting of multiple rectangular nozzles with a synchronized screech instability was studied by Taghavi and Raman[1] and Raman and Taghavi[2,3]. This nozzle showed increased mixing with the jets synchronized. This paper uses the geometry and flow conditions investigated by Raman and Taghavi[3]. The same behavior is shown in a study of the effect of edge tones on multiple jet mixing of high-speed subsonic flows by Krothapalli et. al.[4] using the nozzle described by Krothapalli et. al.[5].

It is proposed that at some point before the jets merge local coherence can be achieved due to external forcing by screech or acoustic feedback and large-scale propagation of instabilities occurs with vary high growth rates. The temporal dynamics produced by the collective interaction of compressible jets is discussed by Miles[6] and the collective interaction of incompressible jets is discussed by Miles[7]. However, the predictions of spatial instability theory have shown better agreement with experiment in free shear flows and jets. Consequently, this paper presents a spatial instability analysis. Following Gaster[8], it is assumed the spatial instability analysis applies in a region where nonlinear effects are small. However, it is acknowledged that the spatially growing linear theory will fail since the amplitude of the disturbance must be bounded.

For single nozzles a reduction in mixing and growth rates with increasing Mach number has been demonstrated experimentally by many investigators[9–13] . Corresponding linear stability analysis of single nozzles shows results that are similar to the experimental studies[14–16] . This is attributed to the fact that eddies are the primary mechanism that transfer energy from the mean flow to the large turbulent structures. However, the following study is based on the idea that these experimental and theoretical results do not apply to the mixing of multiple supersonic rectangular jets with phase locked screech. This paper is based on a linear stability analysis of compressible periodic parallel jet flows which was undertaken to obtain results related to lobed mixer nozzles. In this study, the lobed nozzle design concept is extrapolated in a one dimensional manner to arrive at an array of parallel rectangular nozzles separated by a distance s where the smaller dimension of each nozzle is w_N

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and the longer dimension b is taken to be infinite. Note that it is assumed that even widely spaced rectangular jets which are phase-locked by screech are coherent spatially at some distance from the nozzle. Consequently, in this linear stability analysis it is the collective behavior of compressible periodic parallel jet flow that determines the nozzle interaction.

In this paper, the behavior of the solutions is discussed and the trace of solutions is presented for a range of amplification curves. For each operating condition, only the most highly amplified mode is of interest and special attention is paid to finding a good solution having the maximum amplification. Calculations of spatial growth rates are presented for relative Mach numbers from 1.25 to 1.75 with ratios of nozzle spacing to nozzle width ratios from $s/w_N = 4$ to $s/w_N = 13.7$. The actual values are those for which experimental data is presented by Raman and Taghavi[3].

II. Results

The nozzle configuration is shown in Fig. 1. In this paper, the flow is compressible and the velocity profile is adapted from an equation used by Monkewitz[17] in a study of the absolute and convective instability of two-dimensional wakes. A discussion of the problem formulation is given in Appendix A. Velocity profiles for ratios of nozzle spacing to nozzle width of $s/w_N = 5.5$ and $s/w_N = 7.5$ are shown in Figure 2.

The linear spatial stability analysis is done using Floquet-Bloch theory. It is assumed that in the region of interest a coherent wave can propagate. This type of analysis for temporal stability has been applied by Beaumont[18] to an incompressible flow with a sinusoidal velocity profile perpendicular to the flow and by Miles[6,7] to a compressible and incompressible periodic parallel jet flow. This analysis procedure is discussed in Appendix B.

Stability information is obtained using the flow model described in Appendix A and the Floquet-Bloch method described in Appendix B. In this study of spatially growing waves proportional to $\exp\{\imath(\hat{k}\hat{x} - \hat{\omega}\tau)\}$ where $\hat{k} = kL^*$, $\hat{x} = x/L^*$, $\hat{\omega} = \omega L^*/\Delta U$ and $\tau = t\Delta U/L^*$,the flow disturbance is characterized by a real frequency, $\hat{\omega}$, and a complex relative phase velocity , $\hat{c} = \hat{c}_r + \imath\hat{c}_\imath$ where $\hat{\omega}/\hat{k} = c/\Delta U = \bar{U}/\Delta U + \hat{c}/2$. Consequently, the phase velocity eigenfunction, $\hat{c}_r$, represents the phase velocity scaled by $\Delta U/2$ and shifted by $\bar{U}$ where $\Delta U = U_2 - U_1$ and $\hat{U} = (U_1 + U_2)/2$ so that for $\hat{c}_r = 1$ the disturbance moves at velocity U_2, for $\hat{c}_r = -1$ the disturbance moves at velocity U_1 and for $\hat{c}_r = 0$ the disturbance moves at velocity $\bar{U}$. For a given value of jet relative Mach number, m_2, a value of the ratio of mean velocity to velocity difference, $\bar{U}/\Delta U$, a ratio of inter jet spacing to rectangular nozzle smallest dimension, (s/w_N), and $\hat{c}_\imath$, a range of $\hat{\omega}$ are studied to determine if a growing disturbance with amplification ,$-\hat{k}_\imath$, characterized by a periodicity parameter $\Gamma_\imath$ and a convective phase velocity $\hat{c}_r$ exists. The computer program first evaluates solutions at one hundred fixed values of $\hat{c}_r$ in the range $-1 < \hat{c}_r < 1$. The wave number is given by $\hat{k} = \hat{\omega}/\left(\bar{U}/\Delta U + \hat{c}_r/2 + \imath\hat{c}_\imath/2\right)$. Positive values of $\hat{c}_\imath$ are used in the range from 0 to 1 at intervals of 0.1, since this produces negative k_i values in a useful range. A solution at a given value $\hat{c}_r$ is tabulated if the calculated value of Γ_r is smaller than $5.E-04$ and the calculated value of $|\delta|$ (defined in Appendix B) is less than 2. A further search is made in the $\hat{c}_r$ region where Γ_r is smallest to find the best value of $\hat{c}_r$. An acceptable solution has Γ_r smaller than $1.E-06$ and the calculated value of $|\delta|$ less than 2.

The reported results at each value of $\hat{\omega}$ are limited to three : no solution, one solution, or two solutions. It is possible that more than two solutions exist. The model was developed to study amplification , $-\hat{k}_i$, over a range of Mach numbers and flow geometries for compressible periodic parallel jet flow when the flow is correlated between the jets.

The stability model is for shock-free supersonic jets where no screech tone exists. However, it does depend on the presence of a large span wise multi-nozzle region where a coherent wave can propagate. In this paper, it is suggested that this region can be created by phase locked screech or edge-tones. Since screech generally occurs within a frequency range where the instability waves are highly amplified, the results from this study are used to explain certain events in screech synchronized multiple jets.

For each condition studied, solutions for a range of $\hat{\omega}$ values at a given value of $\hat{c}_i$ were produced to find the region where the maximum amplification, $-\hat{k}_i$, of the unstable wave occurred. The value of $\hat{c}_i$ used were between 0. and 1. using steps of 0.1. The value of $\hat{\omega}$ used was initialized at -0.005 and incremented by 0.005. In general, blocks of 50 $\hat{\omega}$ points were examined at a one time and the calculation for a particular value of $\hat{c}_i$ was abandoned if the current block of 50 points and the previous block of 50 points had no solutions.

To provide information on the spatial instability solution space, the trace of solutions for calculations of spatial amplification is presented for $\bar{U}/\Delta U = 0.5$, and a range of relative Mach numbers, $m_2 = \Delta U/a_2$, and of and nozzle spacings s/w_n shown in Tables 1 and 2 where the values selected are those for which experimental data is presented by Raman and Taghavi[3].

Figures 3 thru 8 show plots of phase velocity eigenvalue, $\hat{c}_r$, amplification, $\hat{k}_\imath$, periodicity factor, $\Gamma_\imath$ as a function of frequency, $\hat{\omega}$ for $0.1 < c_\imath < 1$ using steps of 0.1 for $c_\imath$. For a given value of $c_\imath$ the trace of points for the spatial stability solutions can be characterized as having two regions. At low frequencies the trace shows region or band where the unstable solutions are continuous. This region is generally followed at higher frequencies by isolated islands of instability.

Parameter values for good solutions at the maximum growth rate for each case are presented in Tables 1 and 2. Values are selected so that Γ_r is smaller than $1.E-06$. Some points in Figure 7b and 8b have larger values of $-k_i$. However, for these points the value of Γ_r was larger than $1.E-06$ and they were rejected. In order to find a good solution a progression of points was examined until an acceptable point with Γ_r is smaller than $1.E-06$ was found.

The nozzle width, $w_N{}^*$, is 0.0069m. The frequency of the instability is given by

$$f_r{}^* = \frac{\hat{\omega}_r m_2 a_0}{(1+\frac{s}{w_N})w_N{}^*}$$

where a_0 is the ambient speed of sound (nominally 333 m/sec). Values are given in Tables 1 and 2. Also shown in Tables 1 and 2 is the excitation screech frequency, f_s. The predicted instability frequency is about half the excitation frequency.

III. Discussion

A summary of the results of a study of temporal growth rates by Miles[6] is shown in Figure 9. Large growth rates were found for a range of spacings ,s/w_N, and relative Mach numbers, m_2. At larger spacings and higher Mach numbers the temporal growth rate is reduced.

Figure 10 shows spatial growth rates. This paper shows similar trends when the spatial growth rate is calculated.

IV. Concluding Remarks

A linear instability model for a large span wise multi-nozzle region far downstream where a coherent wave can propagate is presented. Multiple supersonic rectangular jets exhibiting phase-locked screech or excited by edge tones may create such a region. The model may explain an increase in mixing observed in multiple jets exhibiting phase locked screech. This work was conducted with the expectation that multi-jets with synchronized screech could provide increased mixing and reduced noise.

It might be that phase locked screech or edge tones can provide a confining mechanism which produces spatial coherence just as neighboring jets provided a confining mechanism in the experiments of Villermaux and Hopfinger[19] and Villermaux, Gagne, and Hopfinger[20].

The model may be of significant scientific and engineering value in the quest to understand and construct supersonic mixer-ejector nozzles.

Appendix A: Formulation of the problem

Let $(U(y),0,0)$ be the velocity of a steady plane-parallel flow, where the x-axis is in the direction of the flow and

$$U(y) = \bar{U} + \frac{\Delta U}{2}h(y)$$

where U_1 is the velocity outside the jet, U_2 is the mean centerline jet velocity, $\bar{U} = \frac{U_1+U_2}{2}$, the velocity scale is $\Delta U = U_2 - U_1$, and $h(y)$ is the velocity profile function which varies from -1 to 1.

The flow field is perturbed by introducing wave disturbances in the velocity and pressure with amplitudes that are a function of $\hat{y}$. These disturbances are assumed to be traveling waves that are expressed in dimensionless variables as

$$\begin{aligned}&(\tilde{u},\tilde{v},\tilde{w},\tilde{p})\\ &= (\breve{u}(\hat{y}),\breve{v}(\hat{y}),\breve{w}(\hat{y}),\breve{p}(\hat{y}))\exp\left[i\left(\hat{k}\hat{x}-\hat{\omega}\tau\right)\right].\end{aligned}$$

where $\tilde{u},\tilde{v}$, and $\tilde{w}$ are dimensionless velocities, $\tilde{p}$ is the pressure, τ is the time, $\hat{x}$ is the distance along the jet flow, and $\hat{y}$ is the distance normal to the jet along the row of nozzles. The dimensionless variable used herein are $\hat{x} = x/L^*$, $\hat{y} = y/L^*$, and $\tau = t\Delta U/L^*$

The following quantities are also non-dimensionalized by a length scale, L^* and a velocity scale, ΔU.

$$\begin{aligned}\hat{k} &= kL^*,\\ \hat{\omega} &= \frac{\omega L^*}{\Delta U},\\ \frac{\hat{\omega}}{\hat{k}} &= \frac{\omega}{k\Delta U} = \frac{c}{\Delta U} = \breve{c},\end{aligned}$$

and we define $\hat{c}$ as follows

$$\breve{c} = \frac{c}{\Delta U} = \frac{\bar{U}}{\Delta U} + \frac{\hat{c}}{2}.$$

By definition for spacial instability, the frequency, $\hat{\omega}$, is a real positive number, $\hat{k}$ is a complex number that represents the wavenumber in the x-direction, $-k_i$ is the amplification rate of the disturbance in the x direction. $\hat{c}$ a complex number that represents the relative phase velocity. Since

$$\hat{k} = \frac{\hat{\omega}}{\frac{\bar{U}}{\Delta U} + \frac{\hat{c}_r + i\hat{c}_i}{2}}$$

by keeping $\hat{c}_i$ positive and varying $\hat{c}_r$ between $+1$ and -1 $\hat{k}_i$ is always negative and only spatially growing solutions will be found.

From the equations of motion if nonlinear and viscous terms are neglected one can obtain an equation for the y-component of the perturbation velocity as follows:

$$\breve{v}'' - \breve{v}'(\frac{\hat{T}'}{\hat{T}} + \frac{A'}{A}) - \breve{v}\left[\frac{h''}{(h-\hat{c})} + A\imath\hat{k} - (\frac{\hat{T}'}{\hat{T}} + \frac{A'}{A})\frac{h'}{(h-\hat{c})}\right] = 0 \quad (1)$$

where the primes denote differentiation with respect to $\hat{y}$,

$$A = -\imath\hat{k} + m^2\imath\hat{k}\frac{(h-\hat{c})^2}{4}$$

$$A' = 2m^2\imath\hat{k}\frac{(h-\hat{c})h'}{4}$$

$$m^2 = \frac{m_1^2}{\hat{T}}$$

and from Crocco's Equation[21]

$$\hat{T}(y) = \frac{T(y)}{T_1} = \frac{T_2}{T_1} + \frac{(1+h(y))}{2}(1-\frac{T_2}{T_1}) - \frac{1}{2}(m_1)^2(\gamma-1)\frac{(h(y)+1)(h(y)-1)}{4}$$

where

$$m_2 = \frac{\Delta U}{a_2}$$

$$m_1 = \frac{\Delta U}{a_1} = \frac{\Delta U}{a_2}\frac{\sqrt{T_2}}{\sqrt{T_1}} = m_2\frac{\sqrt{T_2}}{\sqrt{T_1}}.$$

where a_1 is the local velocity of sound outside the jet, a_2 is the local velocity of sound inside the jet, T_1 is the local temperature outside the jet, and T_2 is the local temperature inside the jet.

In this paper, the velocity profile function, h(y), is periodic such that

$$h(y+2\pi) = h(y).$$

The velocity profile $h(y)$ is not any exact solution of the Navier-Stokes equation, but it can be considered as a simple model of some real periodic flow.
The velocity profile $h(y)$ discussed herein is given by

$$h(y) = 1 - 2f(y)$$

where the function $f(y)$ is given by

$$f(y) = \frac{1}{\left[1 + \left(\sinh(\frac{\eta}{\sinh(1)})\right)^{18}\right]},$$

$\eta = \Lambda(-1 + \frac{y}{\pi})$. and y goes from 0.0 to 2π. The profile function f(y) is adapted from an equation used by Monkewitz[17] in a study of the absolute and convective instability of two-dimensional wakes. Only two-dimensional disturbances will be considered. A schematic of the nozzle geometry is shown in Figure 1. Velocity profile using $\Lambda = 1.22414$ and $\Lambda = 1.73897$ are shown in Figure 2.

Appendix B: Floquet-Bloch theory

Since the basic flow velocity profile , $f(y)$, is periodic, equation (1) is an example of a Floquet-Bloch problem. The mathematics of solving Floquet-Bloch type problems is discussed by Ince[22], Hochstadt[23], and Zwillinger[24]. Applications to solid state physics are discussed by Sachs[25], Brillouin[26], and Dekker[27]. Applications to spatially periodic flow are discussed by Lorenz[28], Green[29], Beaumont[18], Bai[30] and Gotoh[31,32].
The paper by Beaumont[18] and the description of the Floquet-Bloch theorem by Hochstadt[23] were particularly useful in guiding this research.
A survey of the spatially periodic flow literature is presented by K. Gotoh and M.Y. Yamada[33] .
The second order differential equation can be described by a system of first order differential equations. Let

$$\breve{v} = x_1$$
$$\breve{v}' = x_2$$

so that Eq. 1 can be rewritten as the system

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= Dx_1 + Cx_2 \end{aligned} \quad (2)$$

where

$$C = (\frac{\hat{T}'}{\hat{T}} + \frac{A'}{A})$$

and

$$D = \left[\frac{h''}{(h-\hat{c})} + Ai\hat{k} - (\frac{\hat{T}'}{\hat{T}} + \frac{A'}{A})\frac{h'}{(h-\hat{c})}\right]$$

If $\Phi(y)$ is a fundamental matrix solution of equation ((2)) such that

$$\Phi(0) = \mathbf{I}$$

where $\mathbf{I}$ is the identity matrix, then from the Floquet-Bloch theorem

$$\Phi(y+2\pi) = \Phi(y)\Phi(2\pi)$$

We now introduce two solutions of equation (2) with initial values at $y = 0.0$. We have $\Phi(0) = [\phi_1(0), \phi_2(0)]$ where $\phi(0)_1 = 1$, $\phi'(0)_1 = 0$, $\phi(0)_2 = 0$, and $\phi(0)_2' = 1$. Next we seek the eigenvalues of $\Phi(2\pi)$

$$\begin{aligned} &|\Phi(2\pi) - \mu\mathbf{I}| \\ &= \mu^2 - (\phi_1(2\pi) + \phi_2{}'(2\pi))\mu \\ &\quad +(\phi_1(2\pi)\phi_2{}'(2\pi) - \\ &\quad \phi_2(2\pi)\phi_1{}'(2\pi)) \\ &= \mu^2 - (\phi_1(2\pi) + \phi_2{}'(2\pi))\mu + 1 = 0 \end{aligned} \quad (3)$$

Since

$$\phi_1(2\pi)\phi_2{}'(2\pi) - \phi_2(2\pi)\phi_1{}'(2\pi) = |\Phi(2\pi)| = |\Phi(0)| = 1$$

The independent solutions of equation (2) have the form

$$\phi = X(y)\exp(\frac{log(\mu)}{2\pi}y) = X(y)\exp(\Gamma y)$$

The parameter Γ specifies the period of the eigenfunction ϕ. If Γ is real the eigenfunction grows or decays at infinity. Consequently, only imaginary values of Γ are acceptable. Thus the eigenfunction oscillates in space and is called a continuous mode. The disturbance with $\Gamma_i = 1/n$, where n is a nonzero integer, has a period $2n\pi$. One with $\Gamma_i = 0$ has the same period 2π as the main flow, while an irrational value of Γ_i means the disturbance is aperiodic. Note that the parameter Γ does not appear in the flow equation, but is due to the Floquet-Bloch theorem.

Solutions of 2 are thus of the form

$$X_1(y + 2\pi) = \mu_1 X_1(y)$$
$$X_2(y + 2\pi) = \mu_2 X_2(y)$$

where μ_1 and μ_2 represent the zeros of (3), provided they are distinct.
In general, these solutions will not be periodic.

Conditions for periodic solutions can be found as follows
Let $\mu_1 = e^{i\theta_i}$ and $\mu_2 = e^{-i\theta_i}$.
Then from equation (3)

$$\cos(\theta_i) = \phi_1(2\pi) + \phi_2{}'(2\pi) = \delta/2$$

Consequently, for a solution to be periodic δ must be real and $|\delta|$ smaller than 2.

The constants μ are termed the characteristic multipliers of the Floquet-Bloch system (2) and the corresponding characteristic exponents are determined by the relation $\Gamma = \Gamma_r + i\Gamma_i = \frac{log(\mu)}{2\pi} = \frac{\theta_r}{2\pi} + i\frac{\theta_i}{2\pi}$.

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TABLE I.—PARAMETER VALUES AT MAXIMUM GROWTH RATE FOR m_2 = 1.25, m_2 = 1.35 AND m_2 = 1.45

Parameter	m_2 = 1.25	m_2 = 1.35	m_2 = 1.45
A	1.294735	1.22414	1.73897
s/w_N	4	5.5	7.5
L^*, meters	5.491×10^{-3}	7.1380×10^{-3}	9.3344×10^{-3}
f_s, Hz	12192	10200	8512
f_r^*, Hz	6696	5412	4651
f_r^*/f_s	0.549	0.545	0.546
$\hat{\omega}_r$	0.555	0.54	0.565
$\hat{c}_r$	–0.29605	–0.29936	–2.2653
$\hat{c}_i$	0.4	0.4	0.4
$\hat{k}_r$	1.192	1.1625	1.1527
$\hat{k}_i$	–0.6773	–0.6637	–0.59611
δ_r	0.50985	–0.90483	–1.8499
δ_i	-1.5534×10^{-10}	2.1089×10^{-6}	1.4473×10^{-7}
Γ_i	0.2897	0.17528	6.2044×10^{-2}
Γ_r	-1.2784×10^{-11}	-1.8818×10^{-7}	-3.0307×10^{-8}
$\lvert\delta\rvert$	0.50985	0.90483	1.8499

TABLE II.—PARAMETER VALUES AT MAXIMUM GROWTH RATE FOR m_2 = 1.55, m_2 = 1.65 AND m_2 = 1.75

Parameter	m_2 = 1.55	m_2 = 1.65	m_2 = 1.75
A	1.139382	1.1258595	1.111845
s/w_N	10	11.5	13.7
L^*, meters	1.208×10^{-2}	1.3727×10^{-2}	1.6143×10^{-2}
f_s, Hz	7136	5952	5280
f_r^*, Hz	3876	3344	2183
f_r^*/f_s	0.543	0.562	0.413
$\hat{\omega}_r$	0.57	0.525	0.38
$\hat{c}_r$	–0.105	4.3075×10^{-2}	–0.81295
$\hat{c}_i$	0.4	0.4	1.0
$\hat{k}_r$	1.0617	0.87758	0.13735
$\hat{k}_i$	–0.47449	–0.33654	–0.73431
δ_r	–1.8094	–1.5363	4.9339×10^{-2}
δ_i	8.7953×10^{-8}	8.3042×10^{-8}	1.1×10^{-7}
Γ_i	7.0048×10^{-2}	0.11059	0.24607
Γ_r	-1.6428×10^{-8}	-1.0321×10^{-8}	8.756×10^{-9}
$\lvert\delta\rvert$	1.8094	1.5363	4.9838×10^{-2}

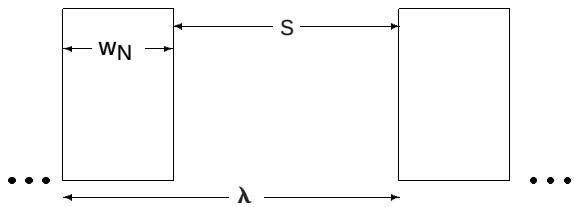

Figure 1.—Nozzle configuration.

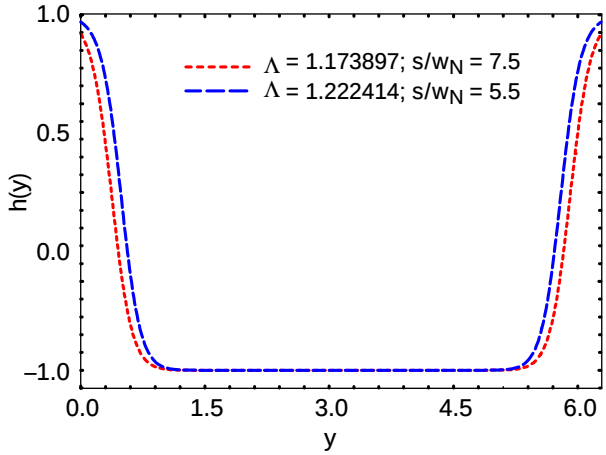

Figure 2.—Typical velocity profile. (Λ = 1.173897; s/w_N = 7.5 and Λ = 1.22414; (s/w_N = 5.5).

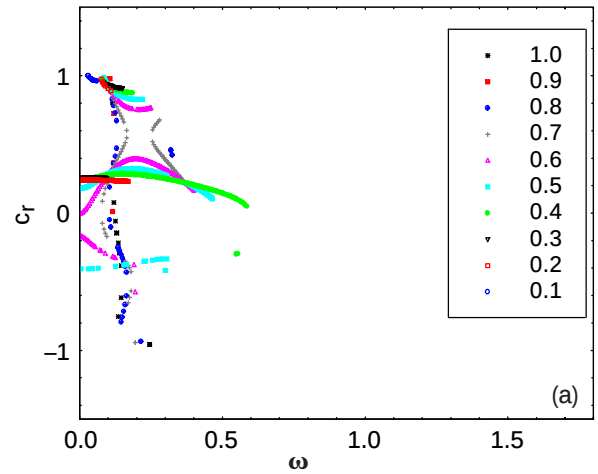

Figure 3.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 4.0$; $\Lambda = 1.294735$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.25$; $T_2 = 1$).

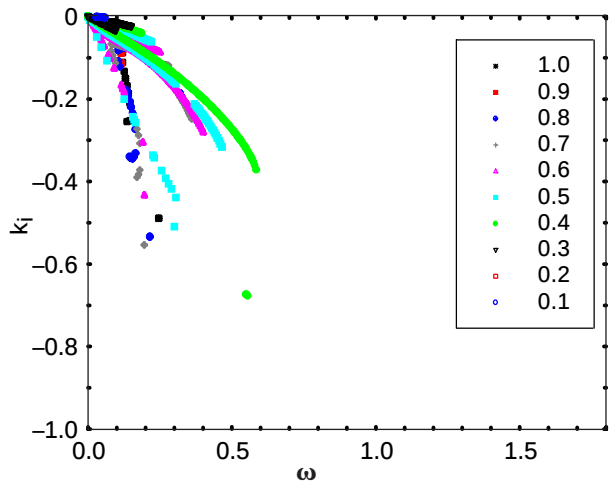

Figure 3.—(b) Amplification, k_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 4.0$; $\Lambda = 1.294735$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.25$; $T_2 = 1$).

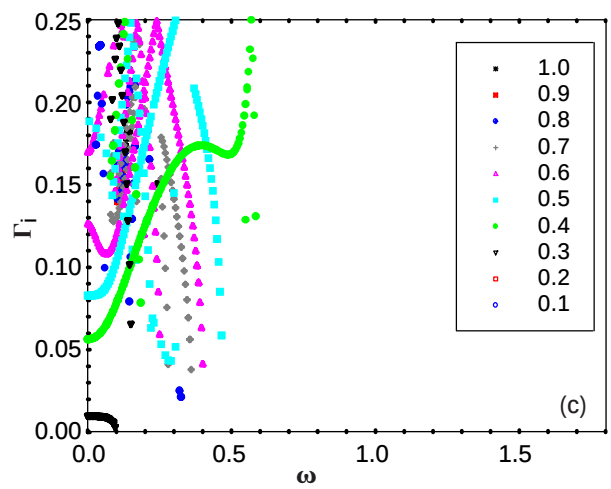

Figure 3.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 4.0$; $\Lambda = 1.4735$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.25$; $T_2 = 1$).

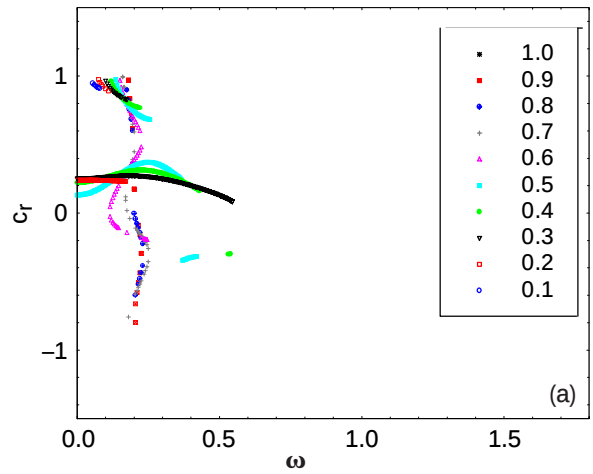

Figure 4.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 0.9$. ($s/w_n = 5.5$; $\Lambda = 1.22414$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.35$; $T_2 = 1$).

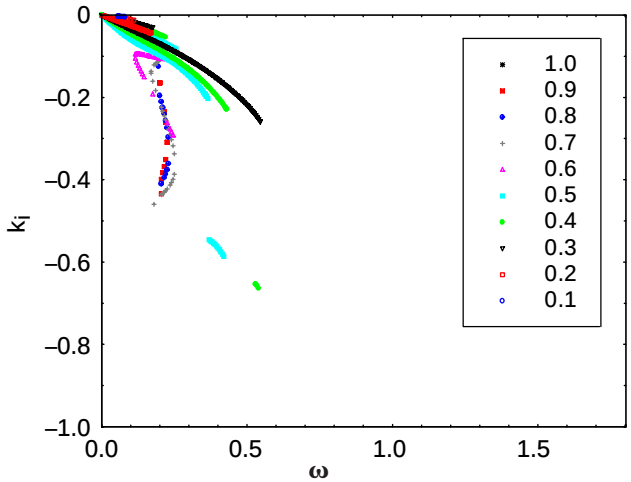

Figure 4.—(b) Amplification, k_i, verses frequency, w, for $0.1 < \hat{c}_i < 0.9$. ($s/w_n = 5.5$; $\Lambda = 1.22414$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.35$; $T_2 = 1$).

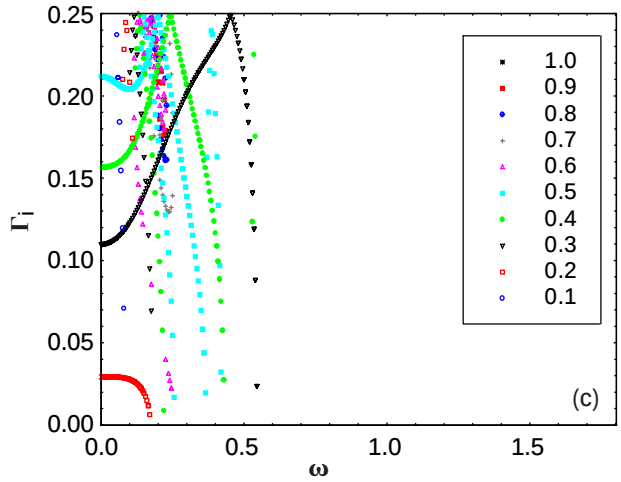

Figure 4.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 0.9$. ($s/w_n = 5.5$; $\Lambda = 1.22414$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.35$; $T_2 = 1$).

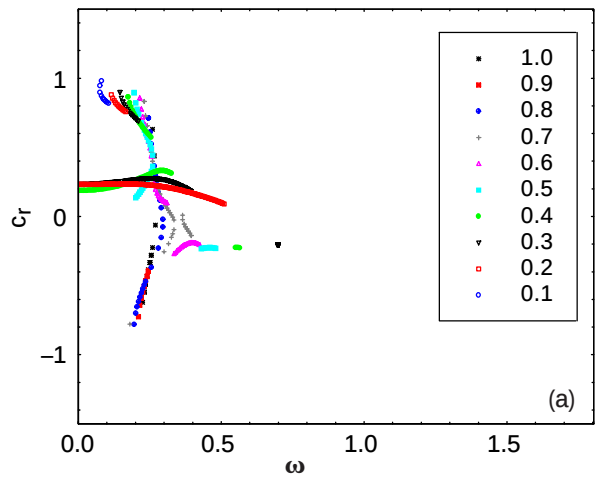

Figure 5.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 7.5$; $\Lambda = 1.173897$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.45$; $T_2 = 1$).

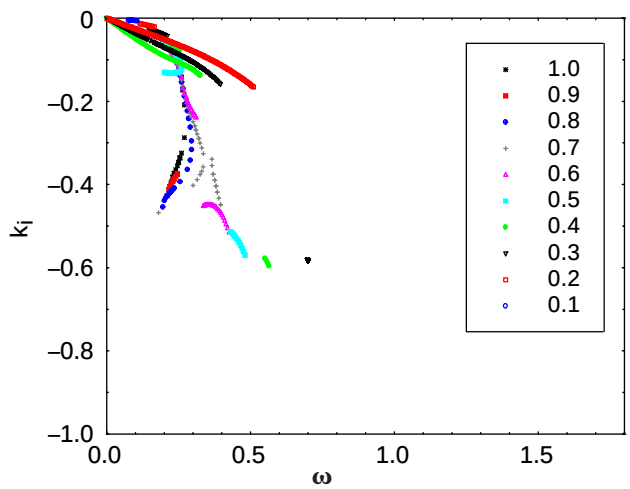

Figure 5.—(b) Amplification, k_i, verses frequency, w, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 7.5$; $\Lambda = 1.173897$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.45$; $T_2 = 1$).

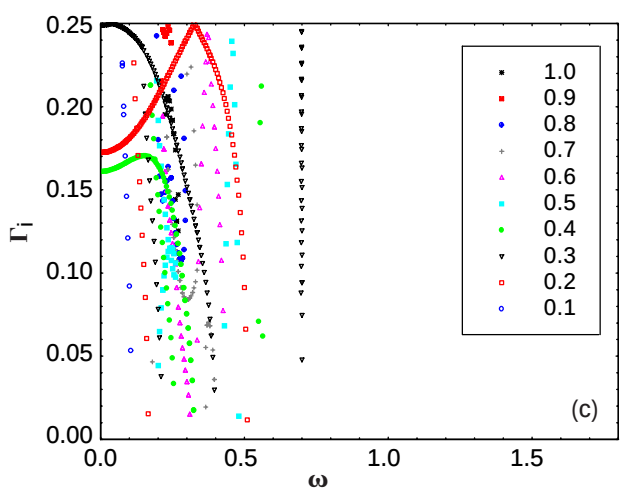

Figure 5.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 7.5$; $\Lambda = 1.173897$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.45$; $T_2 = 1$).

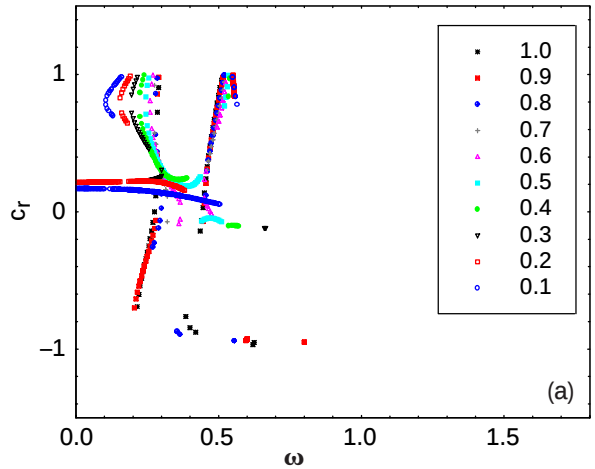

Figure 6.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 10.0$; $\Lambda = 1.139382$; $\tilde{U}/\Delta U = 0.5$; $m_2 = 1.55$; $T_2 = 1$).

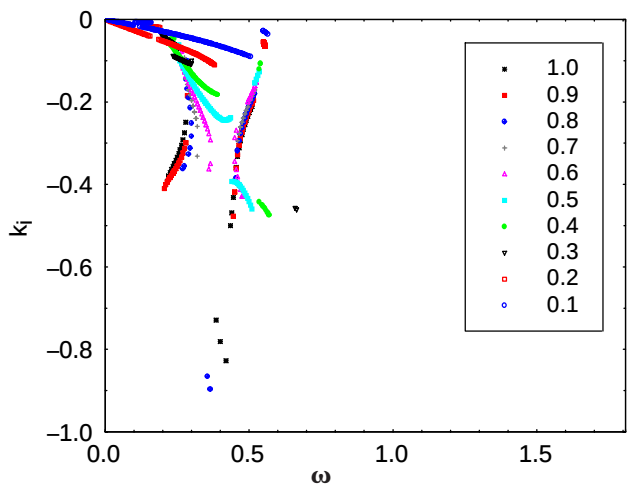

Figure 6.—(b) Amplification, k_i, verses frequency, w, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 10.0$; $\Lambda = 1.139382$; $\tilde{U}/\Delta U = 0.5$; $m_2 = 1.55$; $T_2 = 1$).

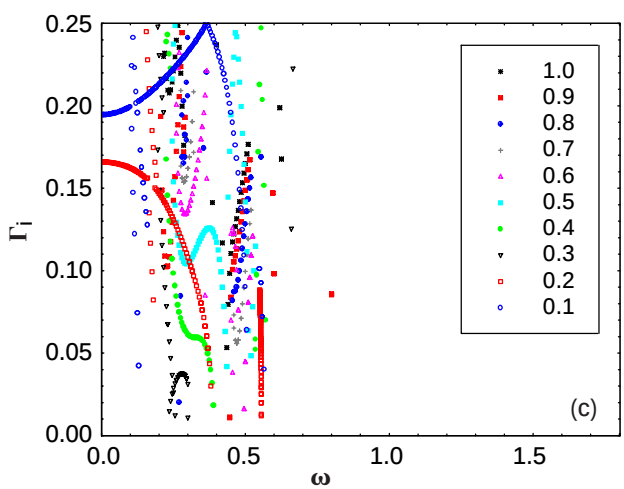

Figure 6.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 10.0$; $\Lambda = 1.139382$; $\tilde{U}/\Delta U = 0.5$; $m_2 = 1.55$; $T_2 = 1$).

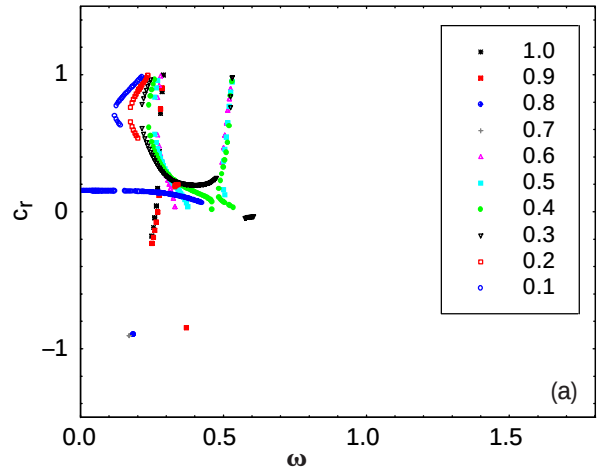

Figure 7.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 11.5$; $\Lambda = 1.1258595$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.65$; $T_2 = 1$).

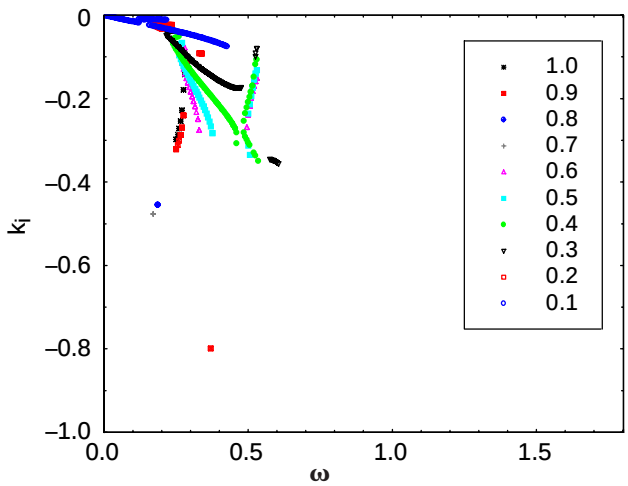

Figure 7.—(b) Amplification, k_i, verses frequency, w, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 11.5$; $\Lambda = 1.1258595$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.65$; $T_2 = 1$).

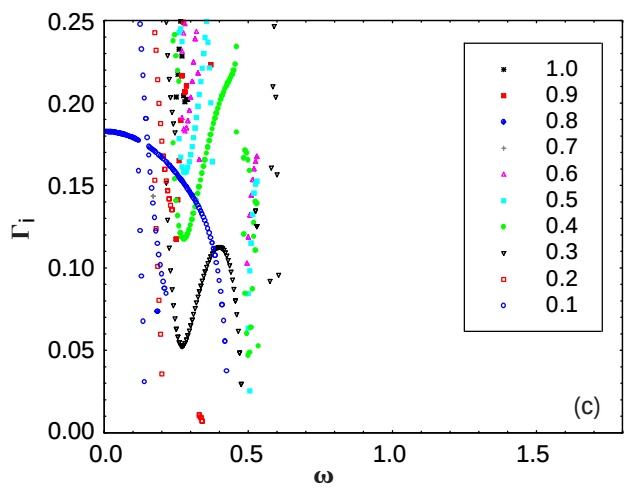

Figure 7.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 11.5$; $\Lambda = 1.1258595$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.65$; $T_2 = 1$).

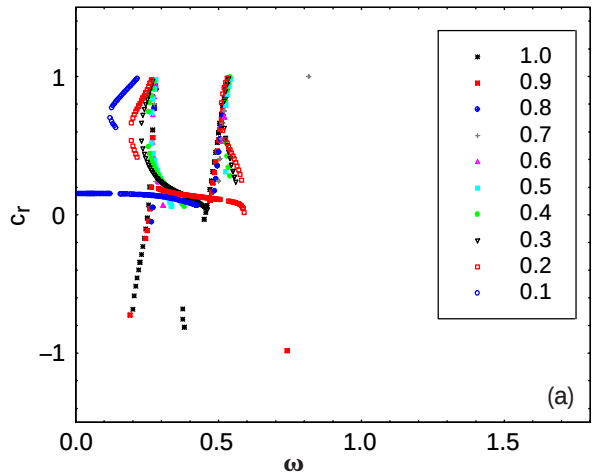

Figure 8.—(a) Phase velocity eigenvalue, c_r, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 11.5$; $\Lambda = 1.111845$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.75$; $T_2 = 1$).

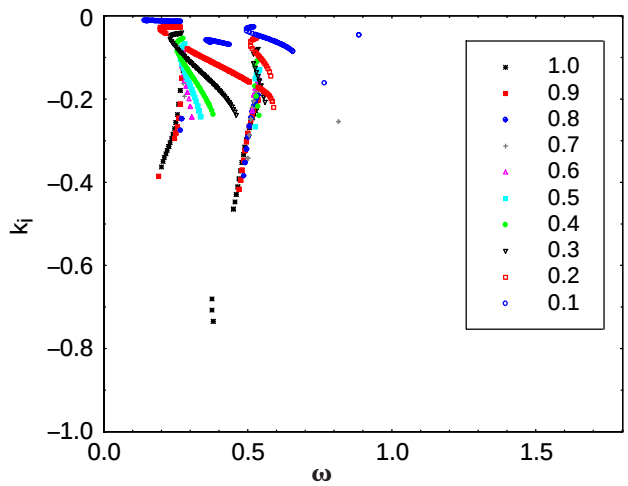

Figure 8.—(b) Amplification, k_i, verses frequency, w, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 11.5$; $\Lambda = 1.111845$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.75$; $T_2 = 1$).

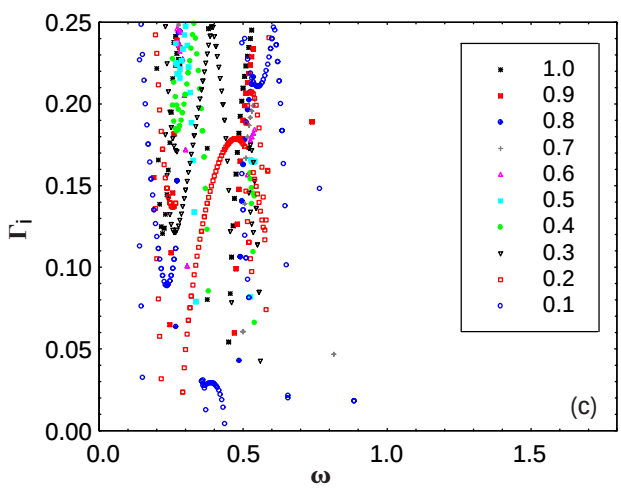

Figure 8.—(c) Periodicity factor, Γ_i, verses frequency, $\hat{w}$, for $0.1 < \hat{c}_i < 1.0$. ($s/w_n = 13.7$; $\Lambda = 1.111845$; $\bar{U}/\Delta U = 0.5$; $m_2 = 1.75$; $T_2 = 1$).

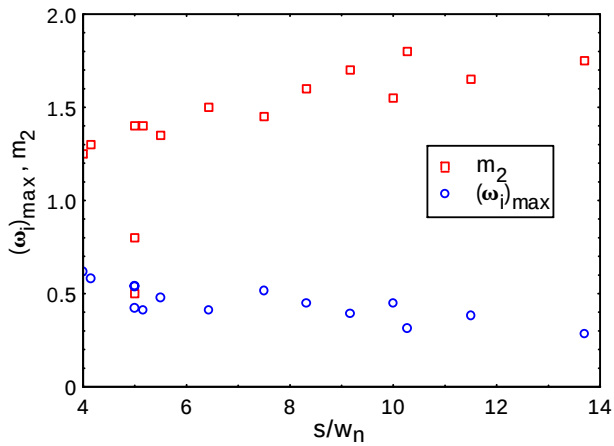

Figure 9.—Temporal growth rate, $(\hat{\omega}_i)_{max}$, results[6] for a range of relative Mach numbers, m_2, and nozzle spacings, s/w_n.

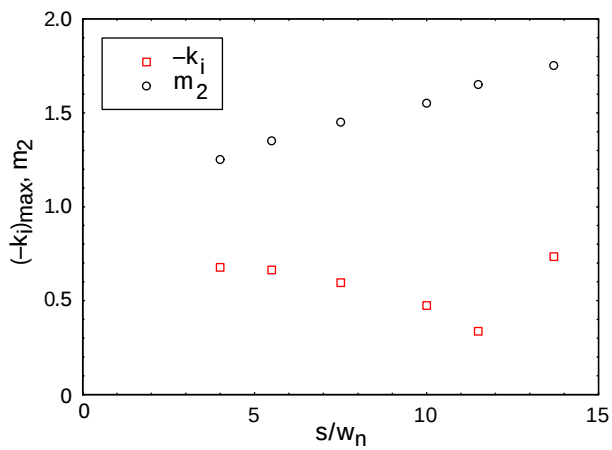

Figure 10.—Spacial growth rate, $(\hat{\omega}_i)_{max}$, results for a range of relative Mach numbers, m_2, and nozzle spacings, s/w_n.

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13. ABSTRACT ***(Maximum 200 words)***

A linear spatial instability model for multiple spatially periodic supersonic rectangular jets is solved using Floquet-Bloch theory. It is assumed that in the region of interest a coherent wave can propagate. For the case studied large spatial growth rates are found. This work is motivated by an increase in mixing found in experimental measurements of spatially periodic supersonic rectangular jets with phase-locked screech and edge tone feedback locked subsonic jets. The results obtained in this paper suggests that phase-locked screech or edge tones may produce correlated spatially periodic jet flow downstream of the nozzles which creates a large span wise multi-nozzle region where a coherent wave can propagate. The large spacial growth rates for eddies obtained by model calculation herein are related to the increased mixing since eddies are the primary mechanism that transfer energy from the mean flow to the large turbulent structures. Calculations of spacial growth rates will be presented for a set of relative Mach numbers and spacings for which experimental measurements have been made. Calculations of spacial growth rates are presented for relative Mach numbers from 1:25 to 1:75 with ratios of nozzle spacing to nozzle width ratios from $s/w_N = 4$ to $s/w_N = 13{:}7$. The model may be of significant scientific and engineering value in the quest to understand and construct supersonic mixer-ejector nozzles which provide increased mixing and reduced noise.

14. SUBJECT TERMS
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