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Effect of Date and Location on Maximum Achievable Altitude for a Solar Powered Aircraft

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Abstract

The maximum altitude attainable for a solar powered aircraft without any energy storage capability is examined. Mission profiles for a solar powered aircraft were generated over a range of latitudes and dates. These profiles were used to determine which latitude-date combinations produced the highest achieavable altitude. Based on the presented analysis the results have shown that for a given time of year lower latitudes produced higher maximum altitudes. For all the cases examined the time and date which produced the highest altitude was around March at the equator.

Symbols

AR	Aspect Ratio
a	Hour Angle (radians)
b	Wing Span (m)
c	Cosine Coefficient
d_n	Day Number Based on the Vernal Equinox
d_{n2}	Day Number Based on Perehelion
e	Earth's Orbital Eccentricity
e_o	Oswald's Efficiency Factor
f	Friction Factor
g	Gravitaional Constant (m/s2)
h	Aircraft Altitude (m)
h_g	Geopotential Altitude (m)
i	Instantanious Time of the Day (hours)
Δi	Time Increment (hours)
m_{tot}	Total Aircraft Mass (kg)
P_a	Power Available (W)
P_{pl}	Payload Power Required (W)
P_{req}	Aircraft Required Power (W)
r	Earth's Mean Raidus (m)
r_{orb}	Orbital Raidus of the Earth (m)
r_{orbm}	Mean Orbital Raidus of the Earth (m)
rc	Rate of Climb (m/s)
S	Sine Coefficient
S_{ff}	Solar Cell Fill Factor
S_{io}	Orbital Solar Intensity (W/m2)
S_{iom}	Mean Orbital Solar Intensity (W/m2)
S_w	Wing Area (m2)
T	Air Temperature (°K)
ϕ	Latitude (Radians)
δ	Declination Angle (Radians)
ρ	Atmospheric Density (kg/m2)
η_{pcon}	Power Conditioning Efficiency
η_{prop}	Propulsion System Efficiency
η_{sc}	Solar Cell Efficiency
τ	Atmospheric Solar Attenuation
θ	Day Angle (Radians)

Introduction

A solar powered aircraft is a unique flying machine from the standpoint that the power available for flight is determined by both the design of the airframe and the local environment. High altitude long endurance solar powered aircraft that have the capability for weeks or months of continuous flight have been shown to be possible at certain latitudes and times of the year.[1,2] Thus the location and date of the flight determine the aircraft's mission capabilities. The energy storage technology required by these aircraft, such as light weight, high power density fuel cells, is not presently available. However, today's solar aircraft, which do not have energy storage, are even more restricted in their mission capability. The available power is dependent not only on the factors of wing area (solar cell area), cell performance, latitude and time of year but also the time of day. These factors can be expressed as a "power profile" that defines the time-varying power available for flight throughout the day. An example of this is shown in figure 1. From this figure it is evident that the available power for the aircraft can vary significantly for different flight locations and dates.

Due to the significant impact the flight date and location have on the capabilities of a solar powered aircraft the question of where and when to fly in order to accomplish a specific goal has to be examined. Recently with the record breaking high altitude flight of the solar Pathfinder[3,4] the answer to that question, what location and date is optimal in order to achieve the maximum altitude for a solar powered aircraft, would be beneficial to any future record breaking attempts. This analysis examines this question by using a computer model to determine what the maximum achievable altitude is for a aircraft powered by solar cells (no energy storage) over a range of latitudes and flight dates.

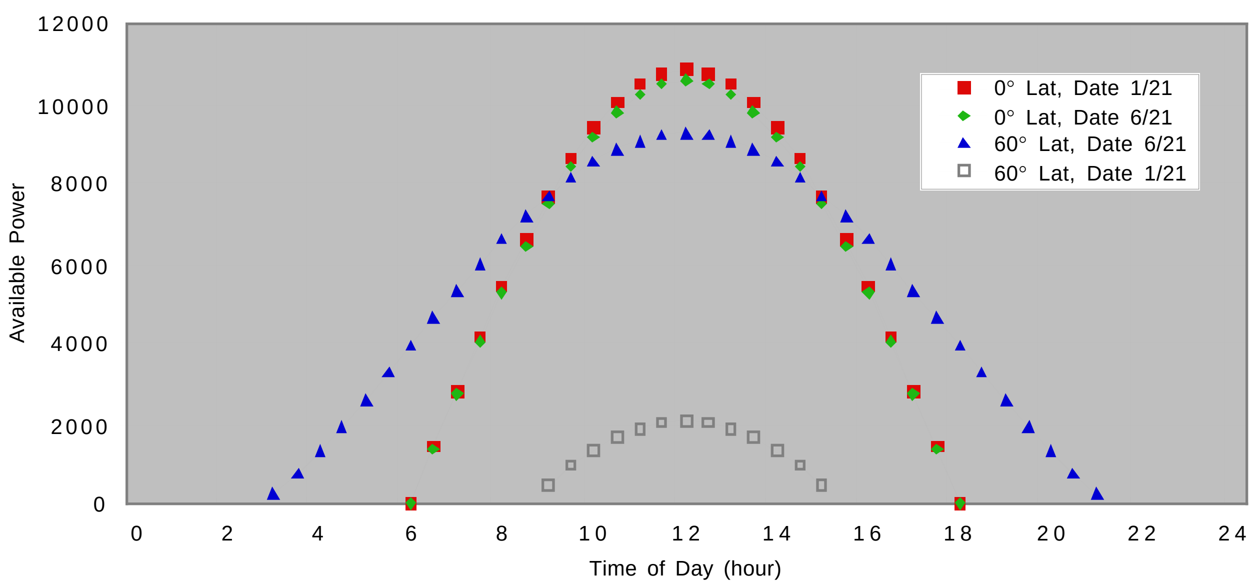

Figure 1 Available Power Curves for Various Location & Date Combinations

Analysis

To accomplish the analysis a computer code was written that would calculate the power required by the aircraft versus power available throughout the day. Based on the available and required power levels an instantaneous rate of climb of the aircraft can be determined for any moment throughout the daytime period. By integrating this instantaneous rate of climb over the daytime period an altitude versus time profile can be generated for the aircraft. The integration is done numerically by incrementing the time of day (Δi). Based on this calculated rate of climb the computer code tracks the aircraft's altitude from takeoff until maximum altitude is reached.

In order for the required and available power for the aircraft to be calculated, a mathematical description of the aircraft and environmental conditions must be assumed. The aircraft is modeled using the descriptions of aircraft performance presented in reference 5 to calculate the power required, while power available is modeled by treating the aircraft as a horizontal flat plate solar collector. Further assumptions used in the analysis are given below.

Solar Cell Efficiency (η_{sc})	14%
Solar Cell Specific Mass	0.25 kg/m^2
Solar Cell Fill Factor (S_{ff})	75%
Power Conditioning Efficiency (η_{pcon})	95%
Propulsion System Efficiency (η_{prop})	85%
Aspect Ratio (AR)	24
Payload Power (P_{pl})	100 W
Payload Mass	100 kg
Oswald's Efficiency Factor (e_o)	0.8
Solar Attenuation Factor(τ)	0.70

Based on these assumptions the power available is calculated as follows.

$$P_a = S_{io}\,\tau\,\eta_{sc}\,S_w\,S_{ff}\,(S - C\cos(-a)) \qquad [1]$$

where

$$S = \sin(\phi)\ \sin(\delta) \qquad [2]$$

$$C = \cos(\phi)\ \cos(\delta) \qquad [3]$$

The latitude (ϕ) and earth's declination angle (δ) vary with the day of the year (d_n). This day number (d_n) is based on the vernal equinox, so d_n = 1 is March 21st.

$$\delta = 0.4091\ \sin(2\ \pi\ d_n / \ 365) \qquad [4]$$

The hour angle (a) is given by the following expression, where “i” is the instantaneous time of day in hours.

$$a = 2 \pi i / 23.935 \quad [5]$$

$$S_{io} = S_{iom} (r_{orbm}^2 / r_{orb}^2) \quad [6]$$

The distance from the earth to the sun (r_{orb}) varies throughout the year. The Earth’s orbital radius (r_{orb})is represented by equations 7 and 8. Where the day number (d_{n2}) is based on the date of perihelion for earth’s orbit. So d_{n2} = 1 is January 4 th.

$$r_{orb} = r_{orbm} (1 - e^2) / (1 + e \cos(\theta)) \quad [7]$$

$$\theta = 2 \pi d_{n2} / 365 \quad [8]$$

The values for the constants used in the above equations are given below.

The mean orbital radius of the Earth (r_{orbm})	1.496×10^8 km
The mean solar intensity at the Earth’s orbital radius (S_{iom})	1352.8 W/m^2
The Earth’s orbital eccentricity (e)	0.017
Acceleration Due to Earth’s Gravity (g)	9.81 m/s
The Earth’s mean radius (r)	6.378E6 m

The power required for the aircraft to maintain level flight is given by equations 9, 10 and 11.

$$P_{req} = 2.4816 (m_{tot} g)^{1.5} f^{0.25} / ((AR \pi e_o S_W)^{0.75} \rho^{0.5} \eta_{prop} \eta_{pcon}) + P_{pl} \quad [9]$$

where

$$f = 0.0117 S_W \quad [10]$$

$$S_W = b^2/AR \quad [11]$$

The total mass of the aircraft, m_{tot}, was calculated from the component mass equations given in reference 2. The air density (ρ) which changes is a function of both altitude above the surface (h in meters) and air temperature (t in °K) is difficult to describe with a single equation over a large change in altitude. In order to get a reasonable approximation of the atmospheric density from the surface up to the maximum altitude for the aircraft, the atmosphere can be broken into four separate regions. A unique atmospheric density equation is used for each region. The relations for each region are given below[6].

For altitudes (h) up to 11 km:

$$t = 288.15 - 0.0065 h_g \quad [12]$$

$$\rho = 1.225 \; (288.15/t)^{-4.256} \qquad [13]$$

For altitudes (h) from 11 km up to 20 km:

$$t = 216.65 \qquad [14]$$

$$\rho = 0.364 \; e^{(11000 - h_g)/6341.62} \qquad [15]$$

For altitudes (h) from 20 km up to 32 km:

$$t = 216.65 + (h_g - 20000)/1000 \qquad [16]$$

$$\rho = 0.088 \; (216.69/t)^{35.16} \qquad [17]$$

For altitudes (h) from 32 km up to 47 km:

$$t = 228.65 + 2.8(h_g - 32000)/1000 \qquad [18]$$

$$\rho = 0.013 \; (228.65/t)^{13.20114} \qquad [19]$$

Where the geopotential altitude (h_g) is given by

$$h_g = r \; h \; /(r+h) \qquad [20]$$

The difference between the available and required power determine the instantaneous rate of climb of the aircraft. This rate of climb is given by the following equation.

$$rc = (P_a - P_{req}) / (m_{tot} \, g) \qquad [21]$$

The rate of climb is used to determine the change in altitude of the aircraft (Δh) over the time increment (Δi). This expression is given below.

$$\Delta h = h + (rc \; \Delta i) \qquad [22]$$

Based on the above equations a computer code was written to perform the analysis. The flow chart for this code is shown in figure 2.

It should be noted that the analysis described above is based on the theoretical calculation of required power which does not take into account all the factors that limit the actual maximum achievable altitude of the aircraft. Factors such as propeller and airframe aerodynamic efficiency variations with altitude due to Reynold's number effects, etc. are neglected. However, these factors do not influence the power generation capability of the aircraft. Therefore the conclusions concerning the effects of location and season on the aircraft's performance are still valid although the actual altitude values calculated are probably somewhat optimistic.

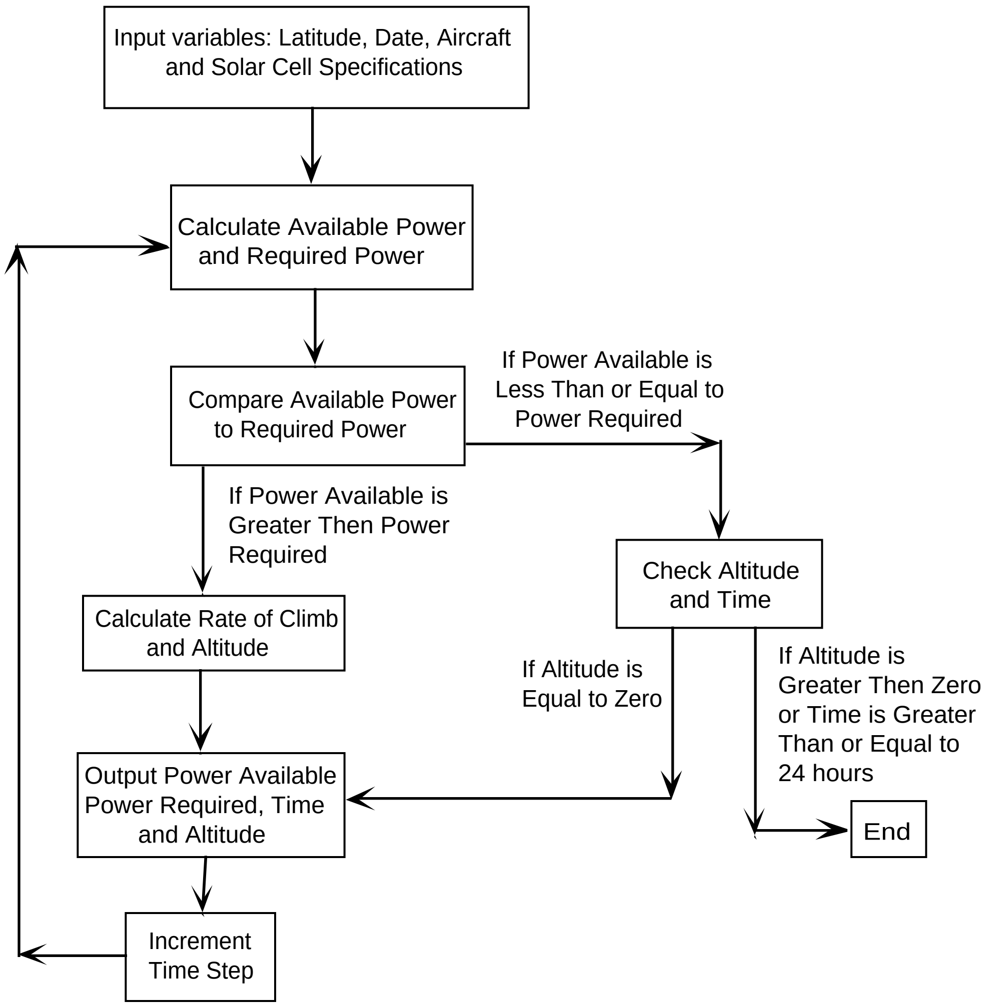

Figure 2 Computer Code Flow Chart

Results

The analysis was used to produce data on the maximum achievable altitude and altitude versus time profiles over a range of location and date combinations. One of the questions that is addressed by these results is whether it is more beneficial to fly at higher latitudes where there are long day periods but relatively low sun angles or to fly at lower latitudes where the day period is shorter but the average sun angle is much greater. The results displayed are only for dates within the first half of the year and latitudes in the northern hemisphere. This is because the results for the second half of the year should mirror those generated for the first half of the year. Similarly data for the southern hemisphere should be the same as that for the northern hemisphere except that the date will be offset 6 months.

Maximum achievable altitude data was produced over the latitude range of 0° to 85° North latitude for each month from January through June. This data was produced using an aircraft wingspan of 50 meters taking into account the assumptions previously listed. The calculated mass for this size aircraft was 435 kg. The results are shown in Table 1 and Figure 3. From this data it can be seen that the maximum achievable altitude for a given latitude generally increases from January through June. And for a given date, as latitude increases, the maximum achievable altitude generally decreases. However, from the equator to about 8° N latitude these trends are reversed. For all the latitude-date combinations that were used, March 21st at 0° latitude produced the highest altitude.

Date	Latitude	Altitude (m)
1/21	0°	27305
2/21	0°	27939
3/21	0°	28013
4/21	15°	27869
5/21	25°	27903
6/21	30°	27937

Table 1 Maximum Altitude and Latitude Location for January through June

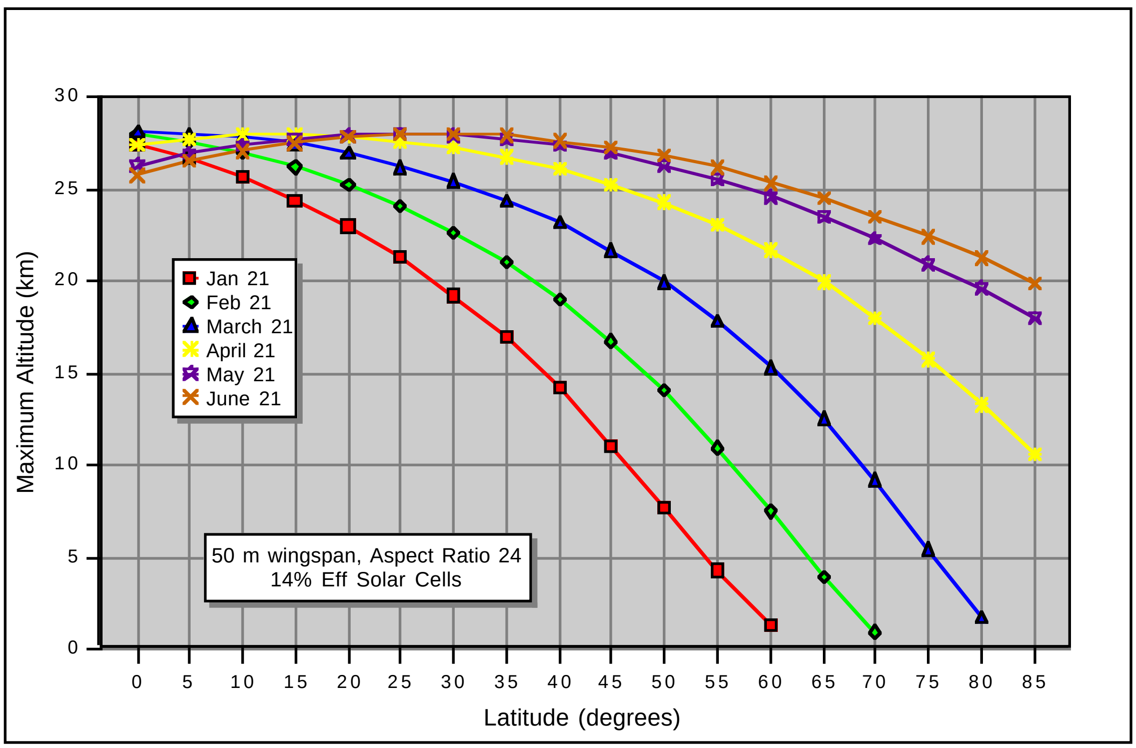

Figure 3 Maximum Altitude vs Latitude for the Months January through June

Profiles of altitude versus day time were also generated over a range of latitudes and dates. These profiles show the aircraft's altitude at intervals from takeoff until maximum altitude is reached. This data is shown in figures 4 through 6 for an aircraft with a 50 meter wingspan. These figures show how the rate of climb and takeoff time is effected by the variation in location and time of year. They also show that the more northern the flight location the more variation in takeoff time and the slower the rate of climb. This would be expected since the more northern the latitude the greater the variation in sunrise time and the lower the average sun elevation angle throughout the day.

In order to determine the effects of aircraft size, data was produced for an aircraft with a wingspan of 70 meters. Beyond the parameters directly related to aircraft size all other assumptions remained the same as those for the 50 m wingspan aircraft. The 70 meter wingspan aircraft mass was calculated to be 571 kg. This data is shown in figures 7 and 8 plotted with similar cases for a 50 meter wingspan aircraft. Figure 7 shows the altitude versus time profiles for both 70 meter and 50 meter wingspan aircraft for the same date and latitude combinations. From this figure it is seen that as the aircraft size increases the maximum altitude also increases, as would be expected. The increase in size also increases the rate of climb and decreases the time to maximum altitude. The effect of location on the rate of climb and takeoff time is fairly consistent for both the 50 meter and 70 meter wingspan cases. Figure 8 shows the maximum achievable altitude over the range of latitudes for both the 70 meter and 50 meter wingspan aircraft. For the June 21st date the difference in maximum altitude between the 70 meter and 50 meter cases is fairly consistent over the complete latitude range. For the January 21st date the increase in altitude between the 70 meter and 50 meter cases decreases as the latitude increases. This indicates that the benefits of a larger aircraft diminish as latitude and the time between the flight date and the summer solstice (June 21st) increases.

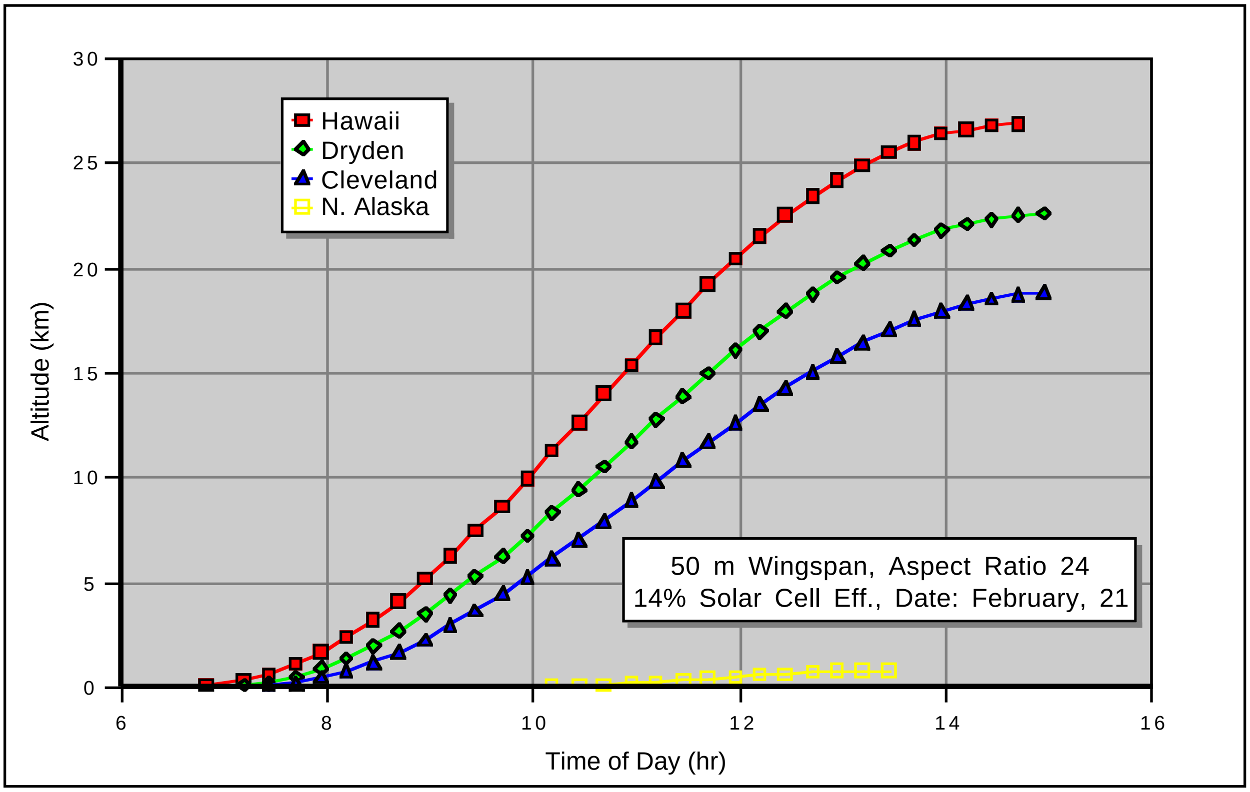

Figure 4 Altitude Profile for February 21st Mission Date

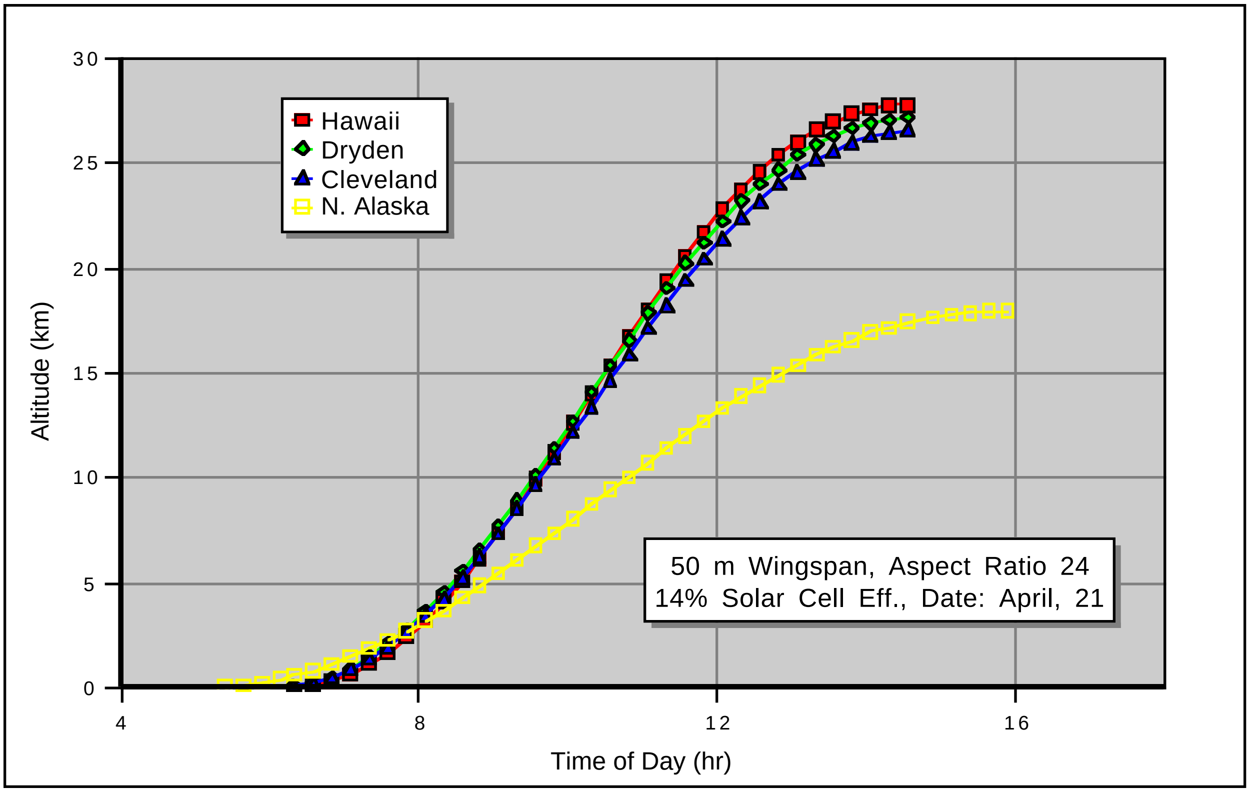

Figure 5 Altitude Profile for April 21st Mission Date

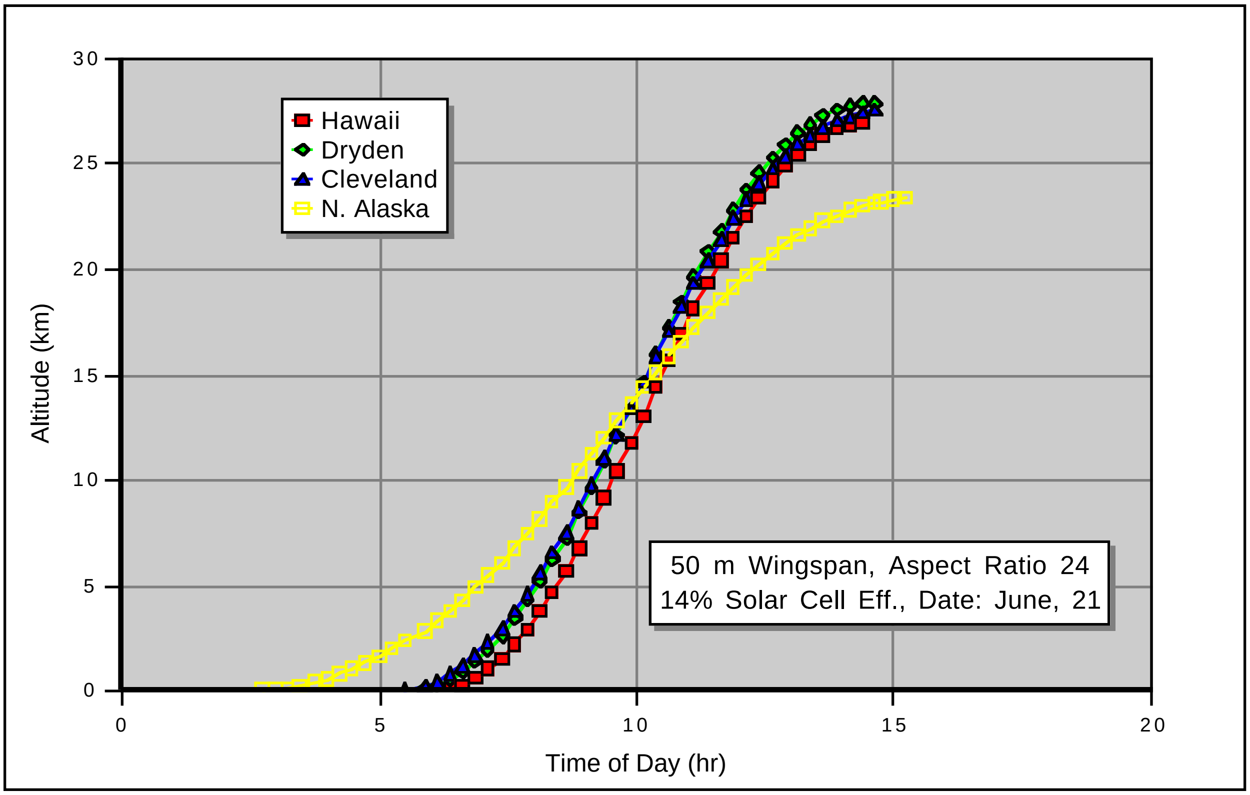

Figure 6 Altitude Profile for June 21st Mission Date

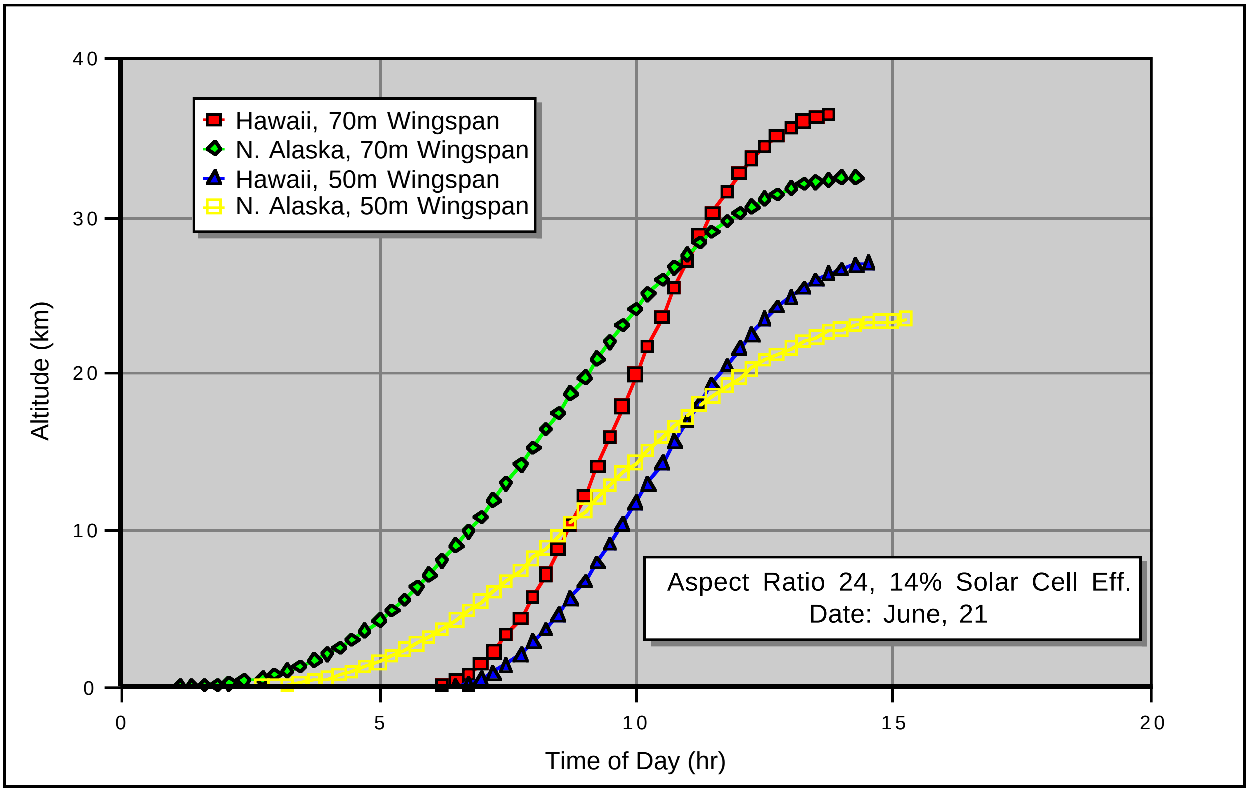

Figure 7 Altitude Profile for 50m and 70m Wingspan Aircraft

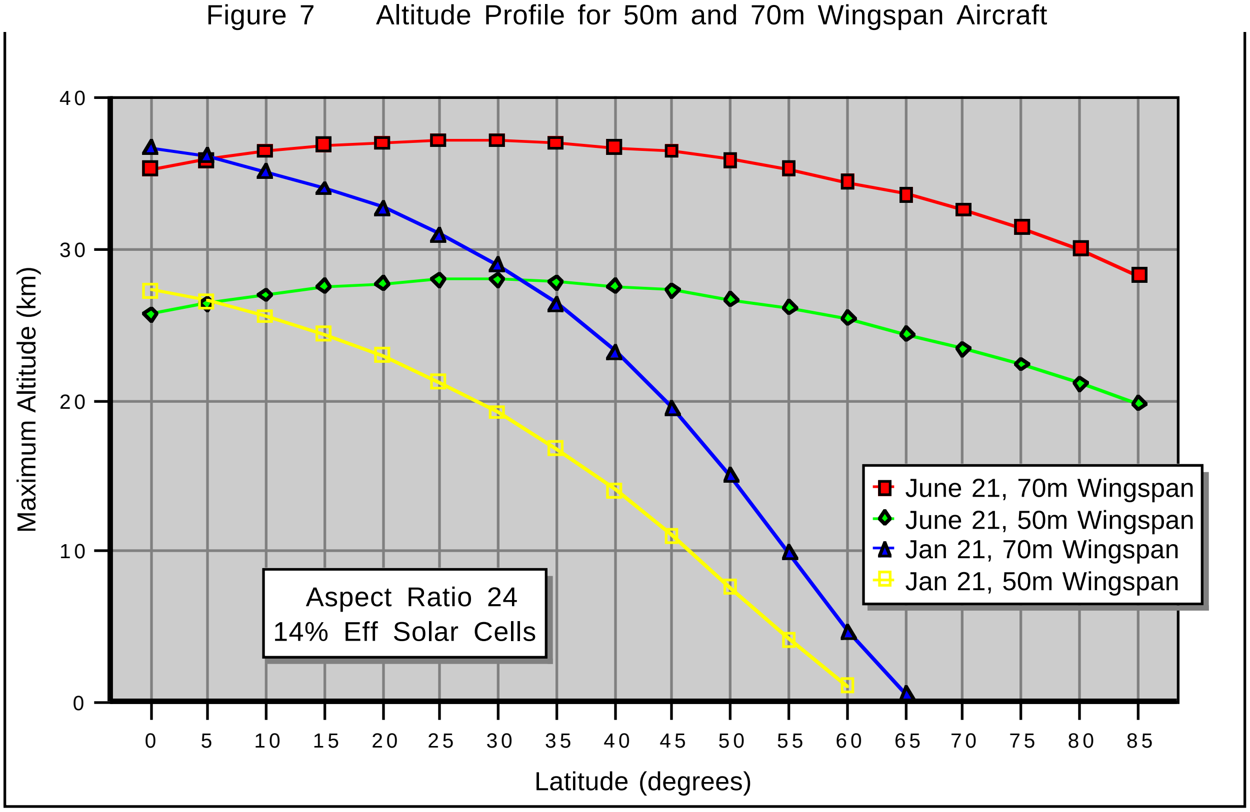

Figure 8 Maximum Altitude for 50m and 70m Wingspan Aircraft

Conclusion

The results from the paper show that for a given sized solar powered aircraft the optimal time and location to fly in order to achieve maximum altitude is around March near the equator. However, it should be noted that the altitude achieved for this date and latitude is not much higher then the altitudes obtained for other dates at their optimum latitudes. Therefore a more general conclusion that can be deduced from the results is that to achieve the highest possible altitude, regardless of the size of the aircraft, it is better to fly at lower latitudes than higher latitudes independent of the time of year. In other words it is better to have shorter day periods and higher sun angles than longer day periods and lower sun angles in order to achieve the highest possible altitude.

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